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Waves in dual-phase-lag thermoelastic materials with voids based on Eringen's nonlocal elasticity

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ABSTRACT

The main idea of the present work is to extend Eringen's theory of nonlocal elasticity to generalized thermoelasticity with dual-phase-lag and voids. Then we study the propagation of time harmonic plane waves in an infinite nonlocal dual-phase-lag thermoelastic medium with voids. Three sets of coupled dilatational waves and an independent transverse wave may travel with distinct speeds through the medium. All these waves are found to be dispersive in nature. The coupled dilatational waves are damping, while the transverse wave is undamped in a certain range of the angular frequency. Coupled dilatational waves are found to be influenced by the presence of voids, thermal field and elastic nonlocal parameter, while the transverse wave is found to be influenced by the nonlocal parameter, but independent of void and thermal parameters. For a particular model, the effects of angular frequency, elastic nonlocality parameter, and some voids and thermal parameters on the wave speeds and damping coefficients of all the propagating waves have been studied numerically. Some comparisons are made between the results obtained for local and nonlocal cases. All the computed results have been depicted graphically and explained in details.

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Introduction

In Eringen's nonlocal elasticity model [1], the stress field at a particular point of an elastic continuum body not only depends on the strain field at that point, but also on the strains at all other points of the body. Hence, the nonlocal continuum theory contains information about long range forces of atoms or molecules and, thus, an internal length scale parameter can be introduced in the formulation. Nonlocal elasticity theories have been applied to the problems of harmonic plane wave propagation in classical and nonclassical elastic materials. To name a few such works are Eringen [2,3] and Roy *et al.* [4] who investigated Rayleigh wave propagation in a rotating nonlocal magnetoelastic half-plane. Narendra [5] studied spectral finite element and nonlocal continuum mechanics based formulation for torsional wave propagation in nano-rods. Chirita [6] discussed thermoelastic surface waves on an exponentially graded half space. Khurana and Tomar [7] studied wave propagation in nonlocal microstretch solid. Rayleigh wave propagation in nonlocal micropolar elastic half-space and in nonlocal elastic half-space with voids has been studied respectively by Khurana and Tomar [8] and Kaur *et al.* [9]. Reflection and refraction of plane waves at a plane interface between two distinct nonlocal micropolar elastic solids half-spaces have

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also been studied recently by Khurana and Tomar [10]. Singh *et al.* [11] studied waves in nonlocal elastic solid with voids. Bachher and Sarkar [12] established a nonlocal theory of thermoelastic materials with voids and fractional derivative heat transfer. They also applied this model to study the interactions in thermoelastic infinite medium with voids due to a time-dependent heat sources. Biswas and Sarkar [13] reported fundamental solution of the steady oscillations equations in porous thermoelastic medium with dual-phase-lag model. Recently, Sarkar and Tomar [14] reported plane waves in nonlocal thermoelastic solid with voids by adopting nonlocal effects in the generalized thermoelasticity with voids [15].

In the present work, the propagation of plane time harmonic waves is investigated in an infinite *nonlocal* dual-phase-lag thermoelastic solid body having void pores based on Eringen's nonlocal elasticity [1]. It has been found that four basic plane waves consisting of three sets of coupled longitudinal (dilatational) waves and one independent transverse wave may travel with distinct speeds in the medium. All these waves are found to be dispersive in nature.

The coupled dilatational waves are damping, while the transverse wave is undamped in a certain range of the angular frequency. Coupled dilatational waves are found to be influenced by the presence of voids, thermal field and elastic nonlocal parameter, while the transverse wave is found to be influenced by the nonlocal parameter only. For a particular model, we highlight the effects of angular frequency, elastic nonlocality parameter, and some void and thermal parameters numerically on the wave speeds and damping coefficients of all the propagating waves. Some comparisons are made between the results obtained for local and nonlocal cases. All the computed results are depicted graphically and discuss.

Field equations and constitutive relations

Within the framework of Eringen's theory of nonlocal elasticity [1], the constitutive relations for thermoelastic solid with voids are given by [12,14]

$$(1-\varepsilon^2\nabla^2)s_{ij} = s_{ij}^L = 2\mu e_{ij} + (\lambda e_{kk} + \beta\phi - \gamma\theta)\delta_{ij}, \tag{1}$$

$$(1-\varepsilon^2\nabla^2)h_i = h_{ij}^L = \alpha\phi_{,i}, \tag{2}$$

$$(1-\varepsilon^2\nabla^2)g = g^L = \tau\dot{\phi} - \xi\phi - \beta e_{kk} + m\theta, \tag{3}$$

$$(1-\varepsilon^2\nabla^2)\rho\eta = (\rho\eta)^L = \gamma e_{kk} + a\theta + m\phi, \tag{4}$$

where the quantities s_{ij}^L, h_i^L, g^L, and $(\rho\eta)^{\mathrm{L}}$ correspond the local thermoelastic solid with voids and δ_{ij} is the Kronecker delta. Other symbols have their usual meanings and borrowed from [12].

The nonlocal generalization of the dual-phase-lag heat conduction law for thermoelastic materials with voids is postulated as (see [12,16–18] for details)

$$(1-\varepsilon^2\nabla^2)\left(\mathbf{q} + \tau_q\dot{\mathbf{q}} + \frac{1}{2}\delta_{1r}\tau_q^2\ddot{\mathbf{q}}\right) = \mathcal{K}\left(1 + \delta_{1r}\tau_T\frac{\partial}{\partial t}\right)\nabla\theta, \tag{5}$$

where $\mathbf{q}$ is the heat flux vector, $\mathcal{K}$ is the thermal conductivity, τT is called the phase-lag of temperature gradient while τq is the phase-lag of heat flux. *Superposed dot* represents temporal derivative. For dual-phase-lag [17,18] thermoelastic model, we put $r=1$ in Eq. (5) and for Lord-Shulman [19] thermoelastic model, $r \neq 1$.

Within the linear theory of thermoelastic material with voids [15], the energy equation has the form (in absence of heat source)

$$\rho T_0\ \dot{\eta} = \nabla\cdot\mathbf{q}, \tag{6}$$

where η and T_0 are respectively the entropy and ambient temperature.

Equations of motion for a nonlocal isotropic thermoelastic solid with voids in absence of the body force and extrinsic equilibrated body force are given by [15]

$$s_{ij,j} = \rho \ddot{u}_i, \tag{7}$$

$$h_{i,i} + g = \rho\chi\ddot{\phi}, \tag{8}$$

where χ is the equilibrated inertia and ρ is the bulk density. A *comma* (,) occurring in the subscript represents the spatial derivative.

Inserting the relations given through (1)–(4), into Eqs. (6)–(8), we obtain the field equations in terms of displacement, volume fraction and temperature for homogeneous and isotropic nonlocal thermoelastic material with voids as

$$\mu\nabla^2\mathbf{u} + (\lambda+\mu)\nabla(\nabla\cdot\mathbf{u}) + \beta\nabla\phi - \gamma\nabla\theta = \rho(1-\varepsilon^2\nabla^2)\ddot{\mathbf{u}}, \tag{9}$$

$$\alpha\nabla^2\phi - \xi\phi - \tau\dot{\phi} - \beta\nabla\cdot\mathbf{u} + m\theta = \rho\chi(1-\varepsilon^2\nabla^2)\ddot{\phi}, \tag{10}$$

$$\mathrm{K}\left(1+\delta_{1r}\tau_T\frac{\partial}{\partial t}\right)\nabla^2\theta = \left(1+\tau_q\frac{\partial}{\partial t}+\frac{1}{2}\delta_{1r}\tau_q^2\frac{\partial^2}{\partial t^2}\right)\left(\rho C_e\dot{\theta} + \gamma T_0\nabla\cdot\dot{\mathbf{u}} + mT_0\dot{\phi}\right), \tag{11}$$

where $aT_0 = \rho C_e$ and C_e is the specific heat at constant strain.

Wave propagation

Introducing the scalar and vector potentials E and $\mathbf{\Omega}$, respectively through the Helmholtz vector decomposition theorem as

$$\mathbf{u} = \nabla E + \nabla\times\mathbf{\Omega}, \qquad \nabla\cdot\mathbf{\Omega} = 0, \tag{12}$$

and plugging these into Eqs. (9)–(11), we obtain the following equations as

$$\beta\phi + (\lambda+2\mu)\nabla^2 E - \gamma\theta - \rho(1-\varepsilon^2\nabla^2)\ddot{E} = 0, \tag{13}$$

$$\mu\nabla^2\mathbf{\Omega} - \rho(1-\varepsilon^2\nabla^2)\ddot{\mathbf{\Omega}} = 0, \tag{14}$$

$$\alpha\nabla^2\phi - \xi\phi - \tau\dot{\phi} - \beta\nabla^2 E + m\theta - \rho\chi(1-\varepsilon^2\nabla^2)\ddot{\phi} = 0, \tag{15}$$

$$\mathrm{K}\left(1+\delta_{1r}\tau_T\frac{\partial}{\partial t}\right)\nabla^2\theta = \left(1+\tau_q\frac{\partial}{\partial t}=\frac{1}{2}\delta_{1r}\tau_q^2\frac{\partial^2}{\partial t^2}\right)\left(\rho C_e\dot{\theta} + \gamma T_0\nabla^2\dot{E} + mT_0\dot{\phi}\right). \tag{16}$$

Note that Eqs. (13), (15), and (16) are coupled through the quantities E, θ, and ϕ, while Eq. (14) is uncoupled. To seek the plane harmonic wave solutions of Eqs. (13)–(16) propagating in the positive direction of a unit vector $\mathbf{n}$ with speed c, we take the form of various potentials as [11,20–22]

$$\{\phi, \theta, E, \mathbf{\Omega}\} = \{A_\phi, A_\theta, A_E, \mathbf{B}\}\exp\{\imath k(\mathbf{n}\cdot\mathbf{r} - ct)\}, \tag{17}$$

where A_ϕ, A_θ, and A_E are constant amplitudes, which may be complex, $\imath = \sqrt{-1}$ is imaginary number, $\mathbf{B}$ is a vector constant, $\mathbf{r}$ $(= x\hat{\imath} + y\hat{\jmath} + z\hat{k})$ is the position vector and k is the real wavenumber. The quantities k and c are connected with angular frequency ω through the relation $\omega = kc$. Moreover, in view of dissipative character of dual-phase-lag thermoelastic theory, we consider c as

$$c = \Re(c) + \imath\Im(c). \tag{18}$$

For the waves to be physically realistic, we should have

$$\Re(c) \geq 0 \text{ and } \Im(c) \leq 0, \tag{19}$$

Here $\Re(c) \geq 0$ gives the wave speed, while $\Im(c) \leq 0$ describes the damping in time of the corresponding propagating wave. Also we note that,

- $\Im(c) = 0$ gives undamped wave in time;

- $\Im(c) < 0$ corresponds to a damped wave in time, decaying exponentially like $\exp[k\Im(c)t]$ to zero as time $t \to \infty$;
- $\Re(c) = 0$ together with $\Im(c) < 0$ generates a standing damped wave in time whose amplitude decays exponentially with time t.

Plugging the expressions of various entities given in (17) into Eqs. (13), (15), and (16), we obtain

$$\beta_1 A_\phi + \left[\omega^2 - \left(c_l^2 - \varepsilon^2\omega^2\right)k^2\right]A_E - \gamma_1 A_\theta = 0, \tag{20}$$

$$\left[\omega^2 - \xi_1 + \imath\tau_1\omega - \left(\alpha_1 - \varepsilon^2\omega^2\right)k^2\right]A_\phi + \beta_2 k^2 A_E + m_1 A_\theta = 0, \tag{21}$$

$$\varepsilon_2\omega^2 A_\phi - \varepsilon_1\omega^2 k^2 A_E + \left[\omega^2 - \frac{K^*}{\tau^*}k^2(1 - \imath\tau_T\omega\delta_{1r})\right]A_\theta = 0, \tag{22}$$

where

$$c_l^2 = \frac{\lambda + 2\mu}{\rho}, \quad \beta_1 = \frac{\beta}{\rho}, \quad \gamma_1 = \frac{\gamma}{\rho}, \quad \alpha_1 = \frac{\alpha}{\rho\chi}, \quad \xi_1 = \frac{\xi}{\rho\chi}, \quad \tau_1 = \frac{\tau}{\rho\chi}, \quad \beta_2 = \frac{\beta}{\rho\chi},$$
$$m_1 = \frac{m}{\rho\chi}, \quad K^* = \frac{\mathrm{K}}{\rho C_e}, \quad \varepsilon_1 = \frac{\gamma T_0}{\rho C_e}, \quad \varepsilon_2 = \frac{m T_0}{\rho C_e}, \quad \tau^* = \tau_q + \frac{l}{\omega}\left(1 - \frac{1}{2}\tau_q^2\omega^2\right).$$

Here c_l is the speed of classical longitudinal wave. Equations (20)–(22) are the system of homogeneous equations in three unknowns, namely, A_ϕ, A_E, and A_θ. For a nontrivial solution of these equations, the determinant of the coefficient matrix must vanish, which yields

$$H_1\ c^6 + H_2\ c^4 + H_3\ c^2 + H_4 = 0, \tag{23}$$

where

$$H_1 = m_1\varepsilon_2 - \omega^2 + \xi_1 - \imath\tau_1\omega, \quad H_4 = \frac{K^*}{\tau^*}\omega^2(1 - \imath\tau_T\omega\delta_{1r})\left(\alpha_1 - \varepsilon^2\omega^2\right)\left(c_l^2 - \varepsilon^2\omega^2\right),$$
$$H_2 = \omega^2\left(\alpha_1 - \varepsilon^2\omega^2\right) - \left(c_l^2 - \varepsilon^2\omega^2\right)H_1 + (m_1\varepsilon_2 - H_1)\left(\frac{K^*}{\tau^*}(1 - \imath\tau_T\omega\delta_{1r}) + \gamma_1\varepsilon_1\right)$$
$$+ \beta_1\beta_2 + m_1\beta_1\varepsilon_1 + \beta_2\gamma_1\varepsilon_2,$$
$$H_3 = -\left(\alpha_1 - \varepsilon^2\omega^2\right)\left[\omega^2\left(c_l^2 - \varepsilon^2\omega^2\right) + \gamma_1\varepsilon_1\right]$$
$$- \frac{K^*}{\tau^*}(1 - \imath\tau_T\omega\delta_{1r})\left[\omega^2\left(\alpha_1 - \varepsilon^2\omega^2\right) + (m_1\varepsilon_2 - H_1)\left(c_l^2 - \varepsilon^2\omega^2\right) + \beta_1\beta_2\right].$$

Equation (23) is the dispersion relation for plane wave propagation in an infinite nonlocal dual-phase-lag thermoelastic medium with voids and provides speeds of propagation of various waves. Note that this equation is cubic in c_2 with complex coefficients, whose roots will provide us the speed of three propagating waves. On solving the dispersion relation (23), one would get six complex roots, that is, $\pm c_j$, $j = 1, 2, 3$. Corresponding to these roots, there exist three sets of coupled longitudinal waves, namely, $P_I -$ wave propagating with speed $\Re(c_1)$, $P_{II} -$ wave propagating with speed $\Re(c_2)$, and $P_{III} -$ wave propagating with speed $\Re(c_3)$. It can be noticed that the values of $c_j's$ are complex, indicating that the corresponding waves are damped in time.

Next, plugging the expression of $\mathbf{\Omega}$ from (17) into (14), we get

$$c_4 = \sqrt{c_t^2 - \varepsilon^2\omega^2}, \tag{24}$$

where c_t $(=\sqrt{\mu/\rho})$ is the speed of classical transverse wave. Relation (24) gives the speed of a transverse wave in the nonlocal thermoelastic medium with voids, which is real for given real value of ω lying in the range $0 < \omega < \omega_c$, $\omega_c = c_t/\varepsilon$. It is clear from the expression of the speed c_4 that the transverse wave speed is independent of thermal and void parameters and travels slower than that in the classical local elastic solid. This reduction in the speed of the transverse wave is due to the presence of nonlocal parameter ε in the thermoelastic material. We also note from (24) that the speed of the transverse wave vanishes at $\omega = \omega_c$. This means that the speed c_4 will remain purely real for $\omega < \omega_c$, zero for $\omega = \omega_c$ and purely imaginary for $\omega > \omega_c$. Thus we can state that the transverse wave is propagating with speed $V_4 = \Re(c_4)$ in the frequency range: $0 < \omega < \omega_c$, and the wave is no more a propagating wave outside the range of the values of the frequency delimited by this frequency range. This shows that $\omega = \omega_c$ acts as a cutoff frequency for the existing transverse wave, a conclusion in accordance with that earlier mentioned by Sarkar and Tomar [14]. This is obviously acceptable since the transverse wave speed does not influenced by the thermal effects. For $\omega > \omega_c$, the transverse wave becomes a standing damped in time wave whose amplitude decays exponentially like $\exp\left(-kt\sqrt{\varepsilon^2\omega^2-c_t^2}\right)$ with time t.

It is clear that $\Im(c_4) = 0$ in the range $0 < \omega \leq \omega_c$, showing that the transverse wave is undamped in time within the range $0 < \omega < \omega_c$. We also note that the longitudinal waves propagating with speeds $V_j = \Re(c_j)$ $(j=1, 2, 3)$, depend upon the angular frequency ω and hence all the existing waves are dispersive in nature. Moreover, it is interesting to note that the transverse wave also depends on ω, showing that the transverse wave is also dispersive in nature due to the presence of the elastic nonlocality in the medium while it is nondispersive in nature in case of local ($\varepsilon = 0$) medium. The presence of nonlocality parameter ε in the expressions of all the wave speeds shows that these waves are influenced by the nonlocality of the medium.

In order to investigate the nature of dilatational waves, inserting the expression of the potential E from (17) into (12), we obtain the displacement vector $\mathbf{u}$ as

$$\mathbf{u} = \imath k b_1 \mathbf{n} \exp\left\{\imath k(\mathbf{n}\cdot\mathbf{r} - c_t)\right\}, \tag{25}$$

which shows that the displacement vector $\mathbf{u}$ is parallel to the vector $\mathbf{n}$. Thus, the particle motion associated with the potential E is in the direction of wave propagation. Hence, the waves propagating with speeds V_j $(j=1, 2, 3)$ are all *longitudinal* in nature as the potentials E, θ, and ϕ are all coupled through the relations (20)–(22). Similarly, inserting Ω from (17) into (14), we notice that the particle motion associated with the potential Ω is normal to the direction of wave propagation $\mathbf{n}$ and consequently the corresponding wave propagating with speed V_4 is *transverse* in nature.

Special cases

Local thermoelastic solid with voids

If the nonlocality effect is neglected from the medium, then we shall be left with thermoelastic medium with voids only. Substituting $\varepsilon = 0$ in (23), we get

$$\tilde{A}c^6 + \tilde{B}c^4 + \tilde{C}c^2 + \tilde{D} = 0, \tag{26}$$

where

$$\tilde{A} = m_1\varepsilon_2 - \omega^2 + \xi_1 - \imath\tau_1\omega, \quad \tilde{D} = \frac{K^*}{\tau^*}\alpha_1\omega^2 c_l^2(1 - \imath\tau_T\delta_{1r}),$$
$$\tilde{B} = \alpha_1\omega^2 - c_l^2\tilde{A} + \left(m_1\varepsilon_2 - \tilde{A}\right)\left(\frac{K^*}{\tau^*}(1 - \imath\tau_T\omega\delta_{1r}) + \gamma_1\varepsilon_1\right) + \beta_1\beta_2 + m_1\beta_1\varepsilon_1 + \beta_2\gamma_1\varepsilon_2,$$
$$\tilde{C} = -\alpha_1\left(c_l^2\omega^2 + \gamma_1\varepsilon_1\right) - \frac{K^*}{\tau^*}(1 - \imath\tau_T\omega\delta_{1r})\left[\alpha_1\omega^2 + c_l^2\left(m_1\varepsilon_2 - \tilde{A}\right) + \beta_1\beta_2\right].$$

Equation (26) provides us the speed of propagation of dilatational waves in thermoelastic medium with voids and dual-phase-lag $(r=1)$ as well as for the L-S theory $(r\neq 1)$. Similarly, by setting $\varepsilon = 0$ into Eq. (24), we see that the speed of transverse wave in thermoelastic medium with voids reduces to the classical transverse wave speed, a result recently obtained by Sarkar and Tomar [14] in the relevant medium when $r \neq 1$.

Local thermoelastic solid without voids

In the absence of void and nonlocal effects, the medium will become thermoelastic medium. To achieve this, we substitute $\alpha_1 = \beta_1 = \beta_2 = \xi_1 = \tau_1 = m_1 = \epsilon_2 = \varepsilon = 0$ in the dispersion relation (26) to obtain

$$\tau^* c^4 - d_1 c^2 + c_l^2 K^* (1 - \imath \tau_T \omega \delta_{1r}) = 0, \quad \tau^* \neq 0, \tag{27}$$

where $d_1 = \tau^*(c_l^2 + \gamma_1 \varepsilon_1) + K^*(1 - \imath \tau_T \omega \delta_{1r})$. Here, the speed of coupled dilatational waves are given by

$$2\tau^* c_{1,2}^* = d_1 \pm \sqrt{d_1^2 - 4 c_l^2 K^* \tau^* (1 - \imath \tau_T \omega \delta_{1r})}. \tag{28}$$

For $r \neq 1$, Eq. (28) gives the coupled longitudinal wave velocities for Lord-Shulman thermoelastic model [19] which are recently obtained by Sarkar and Tomar [14].

We have obtained the dispersion relation (23) in the "velocity-frequency" domain for the present study. As a special case of our present work, we now wish to obtain the dispersion relation for wave propagation in the time differential dual-phase-lag thermoelastic medium [17,18] in "velocity-wavenumber" domain as reported by Chirita, Ciarletta, and Tibullo in their notable work [23]. For this purpose, we rewrite Eq. (27) as follows:

$$\frac{\imath}{k}\left(1 - \imath \tau_q k c - \frac{1}{2}\tau_q^2 k^2 c^2\right)\rho c^3 - \left[\frac{\imath}{k}\rho c_l^2\left(1 - \imath \tau_q k c - \frac{1}{2}\tau_q^2 k^2 c^2\right) + \frac{\mathrm{K} c}{C_e}(1 - \imath \tau_T k c)\right] c + \frac{\mathrm{K} c_l^2}{C_e}(1 - \imath \tau_T k c) - \frac{\imath}{k}\frac{\gamma^2 T_0 c}{\rho C_e}\left(1 - \imath \tau_q k c - \frac{1}{2}\tau_q^2 k^2 c^2\right) = 0.$$

On factorization, above equation reduces to

$$\left(\rho c_l^2 - \rho c^2\right)\left[\mathrm{K}(1 - \imath \tau_T k c) - \frac{\imath}{k}\rho C_e c\left(1 - \imath \tau_q k c - \frac{1}{2}\tau_q^2 k^2 c^2\right)\right] - \frac{\imath}{k}\gamma^2 T_0 c\left(1 - \imath \tau_q k c - \frac{1}{2}\tau_q^2 k^2 c^2\right) = 0.$$

Now, if we use the notations of Chirita, Ciarletta and Tibullo [23], that is if we write i, χ, υ, aT_0, k, and β in place of $\imath$, k, c, ρC_e, K, and γ, respectively, then the above equation further simplified to

$$\left(\lambda + 2\mu - \rho \upsilon^2\right)\left[k - i\chi \upsilon k \tau_T - \frac{i}{\chi} a T_0 \upsilon\left(1 - i\chi \upsilon \tau_q - \frac{1}{2}\chi^2 \upsilon^2 \tau_q^2\right)\right] - \frac{i}{\chi}\beta^2 T_0 \upsilon\left(1 - i\chi \upsilon \tau_q - \frac{1}{2}\chi^2 \upsilon^2 \tau_q^2\right) = 0, \tag{29}$$

Equation (29) is exactly same with the Eq. (3.7) obtained by Chirita, Ciarletta, and Tibullo [23]. Furthermore, if we substitute

$$\upsilon = i c_2 w, \; c_1 = \sqrt{\frac{\lambda + 2\mu}{\varrho}}, c_2 = \sqrt{\frac{\mu}{\varrho}}, \; \varepsilon = \frac{\beta^2 T_0}{\varrho a T_0 c_1^2},$$

then the Eq. (29) becomes

$$\left(1 + \frac{c_2^2}{c_1^2} w^2\right)\left[1 + \chi \tau_T c_2 w) + \frac{a T_0 c_2}{\chi k} w\left(1 + \chi \tau_q c_2 w + \frac{1}{2}\chi^2 \tau_q^2 c_2^2 w^2\right)\right] + \frac{\varepsilon a T_0 c_2}{\chi k} w\left(1 + \chi \tau_q c_2 w + \frac{1}{2}\chi^2 \tau_q^2 c_2^2 w^2\right) = 0, \tag{30}$$

which is in complete agreement with the dispersion equation (3.17) as reported by Chirita, Ciarletta, and Tibullo [23]. So, one may obtain the results represented by Chirita and Tibullo [23] from the dispersion relation (30).

Nonlocal thermoelastic solid without voids

If we neglect the void effects only, then we shall be left with nonlocal thermoelastic medium. For this, setting $\alpha_1 = \beta_1 = \beta_2 = \xi_1 = \tau_1 = m_1 = \epsilon_2 = 0$ into Eq. (23), we obtain the following quadratic equation

$$c^4 - \bar{B}c^2 + \bar{C} = 0, \tag{31}$$

where

$$\bar{B} = c_l^2 - \varepsilon^2\omega^2 + \gamma_1\varepsilon_1 + \frac{K^*}{\tau^*}(1 - \imath\tau_T\omega\delta_{1r}), \quad \bar{C} = \frac{K^*}{\tau^*}(1 - \imath\tau_T\omega\delta_{1r})\left(c_l^2 - \varepsilon^2\omega^2\right).$$

In this case, the speeds of the coupled dilatational waves become

$$c_{1,2}^2 = \frac{\bar{B} \pm \sqrt{B^2 - 4C}}{2}, \tag{32}$$

whereas the speed of the transverse wave remains unchanged. These results have been recently obtained by Sarkar and Tomar [14].

Nonlocal elastic solid with voids

If we neglect the thermal effects, that is, for $K^* = \gamma_1 = \epsilon_1 = \epsilon_2 = m_1 = {}_0$, the cubic equation (23) reduces to the following quadratic equation in c_2 as

$$Ac^4 + Bc^2 + C = 0, \tag{33}$$

where

$$A = \rho\left(\rho\chi\omega^2 + \imath\tau\omega - \xi\right), \quad B = 2\rho^2\chi\varepsilon^2\omega^4 + \imath\rho\tau\varepsilon^2\omega^3\left[\chi(\lambda + 2\mu) + \alpha + \varepsilon^2\xi\right] + (\lambda + 2\mu)(\xi - \imath\tau\omega) - \beta^2,$$
$$\mathrm{C} = \rho^2\chi\varepsilon^4\omega^6 - \rho\varepsilon^2[\chi(\lambda + 2\mu) + \alpha]\omega^4 + \alpha(\lambda + 2\mu)\omega^2.$$

Equation (33) is earlier reported by Singh *et al.* [11] for the relevant medium.

Nonlocal elastic solid without voids

If we neglect the thermal and void effects simultaneously from the medium, then we shall be left with an elastic medium having nonlocality only. In this case, the speed of the coupled dilatational waves are obtained from Eq. (32) as

$$c_1 = \sqrt{c_l^2 - \varepsilon^2\omega^2} \quad \text{and} \quad c_2 = 0. \tag{34}$$

and the speed of the transverse wave remains the same as it is independent of thermal and void parameters. We notice that in the absence of thermal and void effects, that is, when only the elastic nonlocality presents in the medium, the square of the speeds of the dilatational and transverse waves are frequency dependent and both reduced by the same amount equal to $\varepsilon_2\ \omega_2$.

Table 1. Numerical values of parameters.

Symbol	Value	Unit	Symbol	Value	Unit
λ	1.5×10^{10}	Pa	μ	7.5×10^{9}	Pa
α	8×10^{9}	Pa m^2	β	10^{10}	Pa
ξ	1.2×10^{10}	Pa m^2	τ	10^{6}	Pa s
χ	0.16	kg/m^3	T_0	300	K
ρ	2×10^{3}	J $kg^{-1}deg^{-1}$	K	0.016×10^{-3}	W $m^{-1}deg^{-1}$
C_e	3×10^{-9}	N m^{-2} deg^{-1}	m	2×10^{6}	N $m^{-2}deg^{-1}$
γ	2.68×10^{6}	s	τT	0.015	s
τ_q	0.02				

Classical elastic solid

In the absence of nonlocality, thermal and void effects from the medium, we note from Eq. (34) that the speed of the coupled dilatational waves reduce to $c_1 = c_l$ and $c_2 = 0$, while the speed of the transverse wave reduces to $c_4 = c_t$. Thus, all the waves of classical elasticity are successfully recovered as was expected beforehand.

Nonlocal non-Voigt thermoelastic solid with voids

A solid body with void pores is said to be non-Voigt if its viscous type behavior is absent, that is, $\tau = 0$. Substituting $\tau = 0$ in the dispersion relation (23), we can obtain the dispersion relation for *nonlocal non-Voigt thermoelastic solid*. In this case, the speed of longitudinal waves will be affected, while the transverse wave is already independent of void parameters, hence unaffected.

Numerical results and discussion

In this section, we perform some numerical computations to study the wave propagation characteristics through a nonlocal dual-phase-lag thermoelastic material with void pores. For this purpose, we have borrowed the values of relevant material parameters from Sarkar and Tomar [14] and Sing *et al.* [11] which are given in Table 1.

Using the Table 1 values, the wave speeds $V_j = \Re(c_j)$ and the damping coefficients $D_j = \Im(c_j)$, where $j = 1, 2, 3, 4$ of the existing waves are computed by solving Eq. (23) for various cases. In our computational work, we made nonlocal parameter dimensionless by defining $\epsilon = \varepsilon/d$, where the value of the parameter d is taken unit nanometer (see [7] for details).

We made some graphical representations for the wave speeds and damping coefficients against the angular frequency ω ranging from 10 to 103 taking $\epsilon = 1.0$ through Figure 1a–f and we can make the following conclusions:

- From Figure 1a, c, e, we observed that the coupled dilatational waves propagate in local dual-phase-lag thermoelastic medium having void pores with lager speed as compared to that in nonlocal medium.
- From Figure 1b, d, f, we found smaller values for damping coefficients in local medium when compared to that in case of nonlocal medium.
- It is also observed that, the wave speeds, in decreasing order, are as follows: V_3, V_1, V_2 whereas for damping coefficients the fact is same but in reverse order. This is expected as wave propagating with larger speed, experiences smaller damping in amplitude.

Another set of Figure 2a–f has been depicted for wave speeds and damping coefficients against the dimensionless nonlocal parameter $(0 \leq \epsilon \leq 5)$ [7].

- In this set of figures, it is interesting to note that elastic nonlocality parameter ϵ diminishes the magnitudes in both the wave speeds and damping coefficients. This makes sense from Eq. (34).

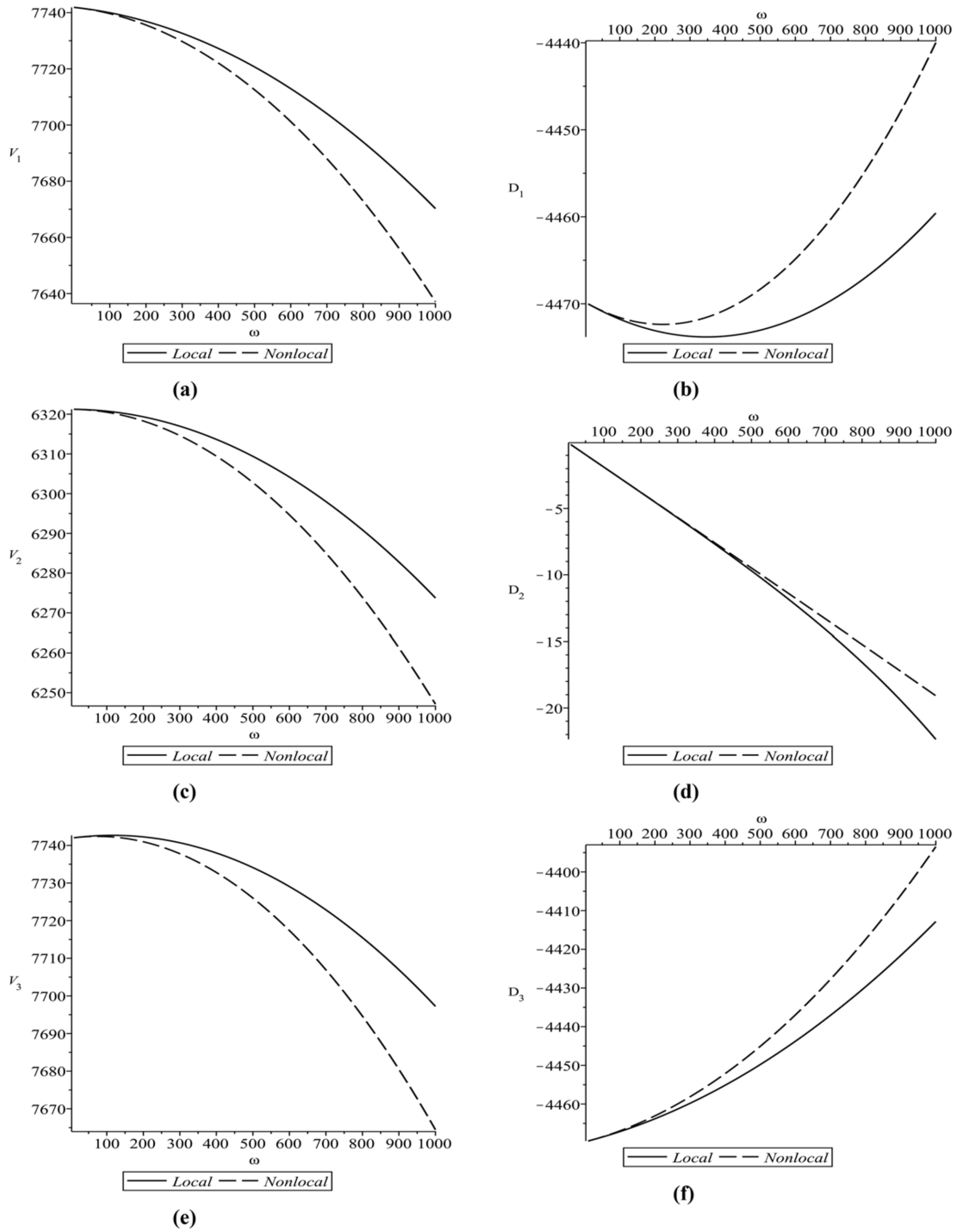

Figure 1. Comparison of wave speeds and corresponding damping coefficients with respect to angular frequency ω for local ($\epsilon = 0$) and nonlocal ($\epsilon > 0$) medium.

- We find larger value in wave speeds and smaller value in damping coefficients for comparatively larger frequency.
- Comparing the wave speeds and corresponding damping coefficients together [i.e., (V_1, D_1); (V_2, D_2); and (V_3, D_3)], it can be noticed that the coupled dilatational waves having smaller damping propagate with larger speed.

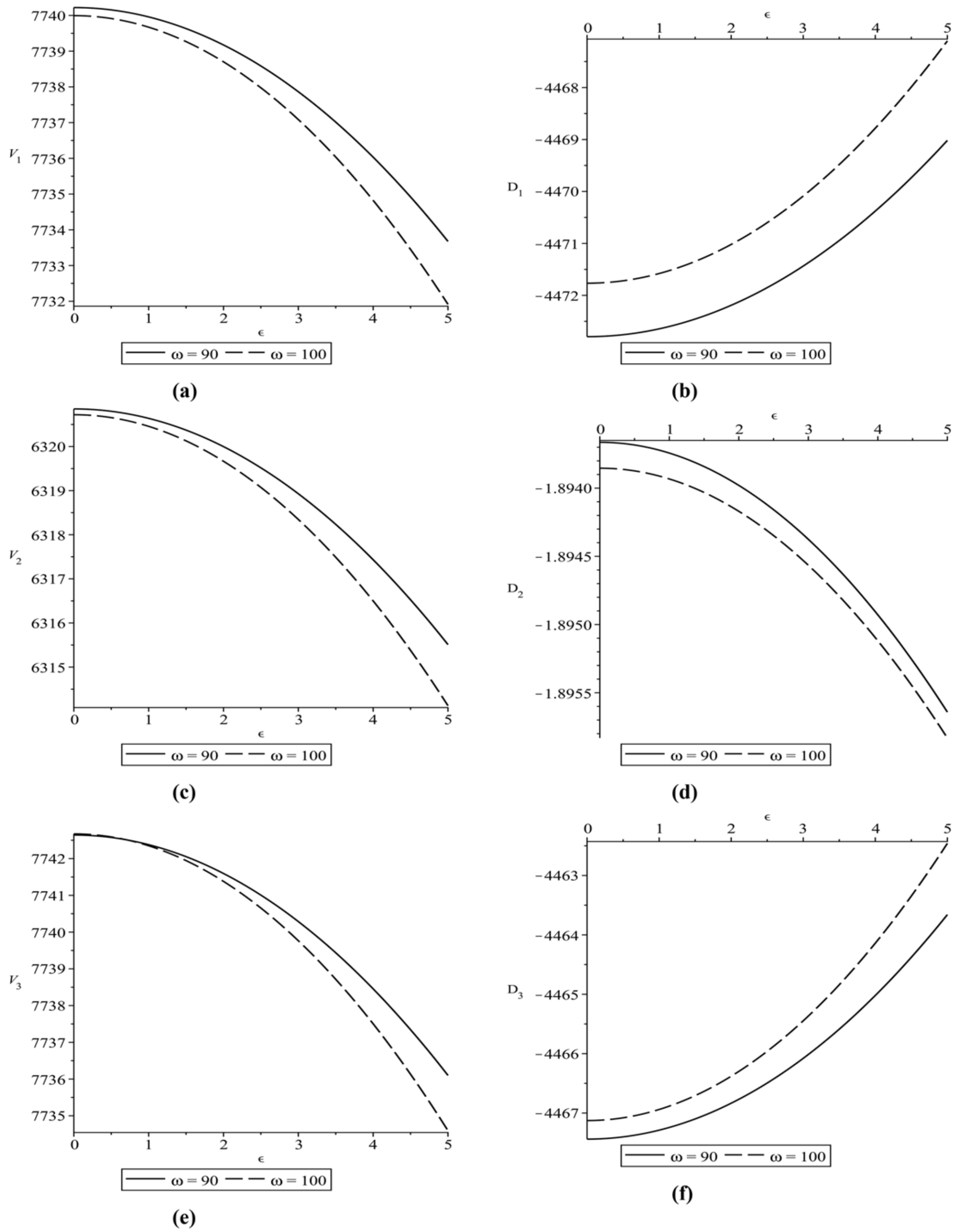

Figure 2. Comparison of wave speeds and corresponding damping coefficients with respect to dimensionless elastic nonlocality parameter ϵ for two different ω.

To exhibit the impact of ω and ϵ simultaneously in both the wave speeds and damping coefficients, we represent some three dimensional graphs through Figure 3a–f. The reverse order phenomena between wave speeds and damping coefficients has been depicted from Figure 3a–f, no matter whatever may be the values of ω and ϵ. All the features discussed in Figures 1a–f and 2a–f can be viewed at the same time here.

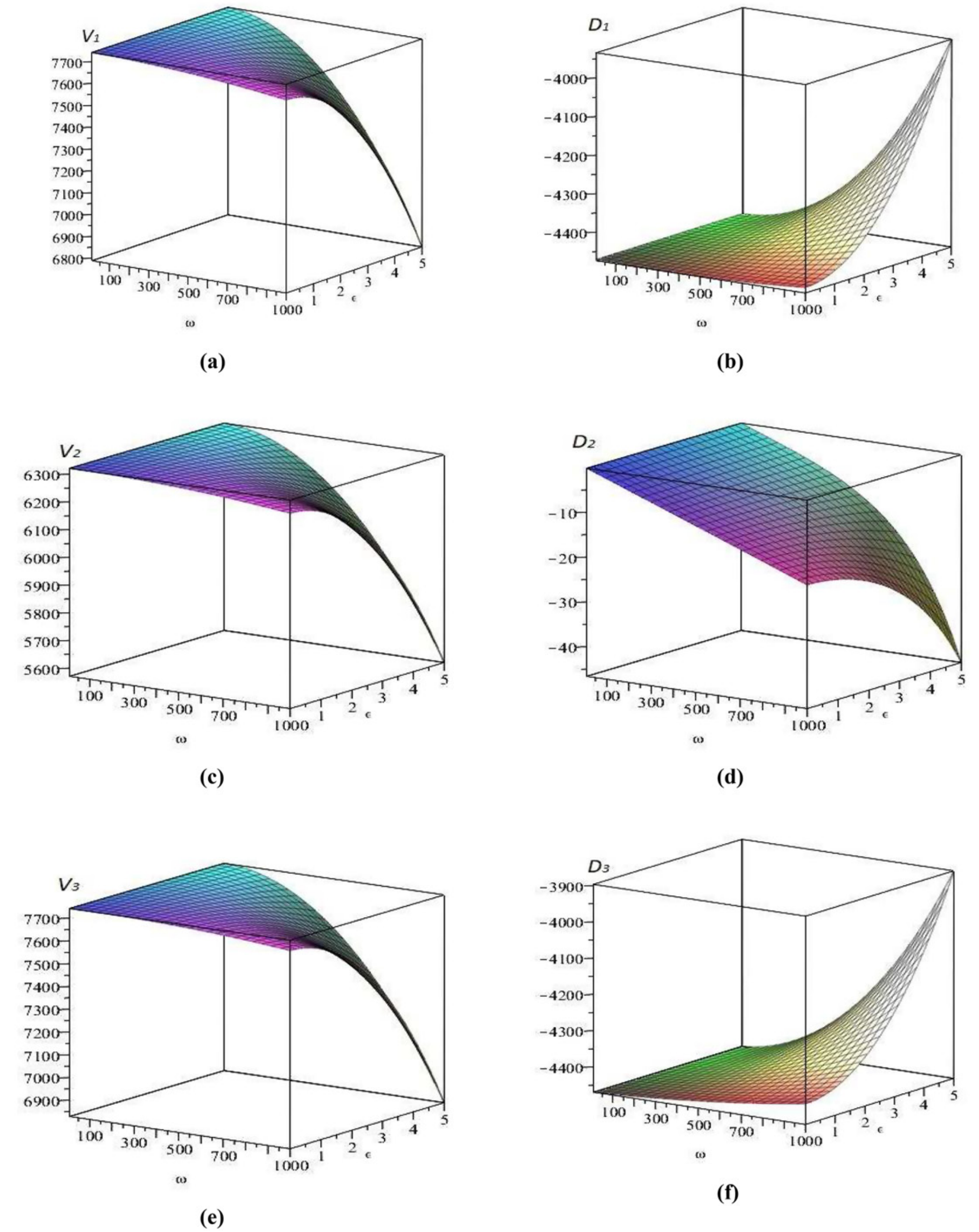

Figure 3. Variations of wave speeds and corresponding damping coefficients with respect to ω and ϵ (dimensionless).

Figure 4a–f is plotted to analyze the influence of various coupling parameter of interest (β, γ, m) on the wave speeds and corresponding damping coefficients.

- The impact of *thermoelastic coupling* parameter γ has been shown in Figure 4a, b for wave speed V_1 and corresponding damping coefficient D_1 for local and nonlocal medium. It is seen that larger magnitudes of γ raise the magnitudes in both V_1 and D_1.

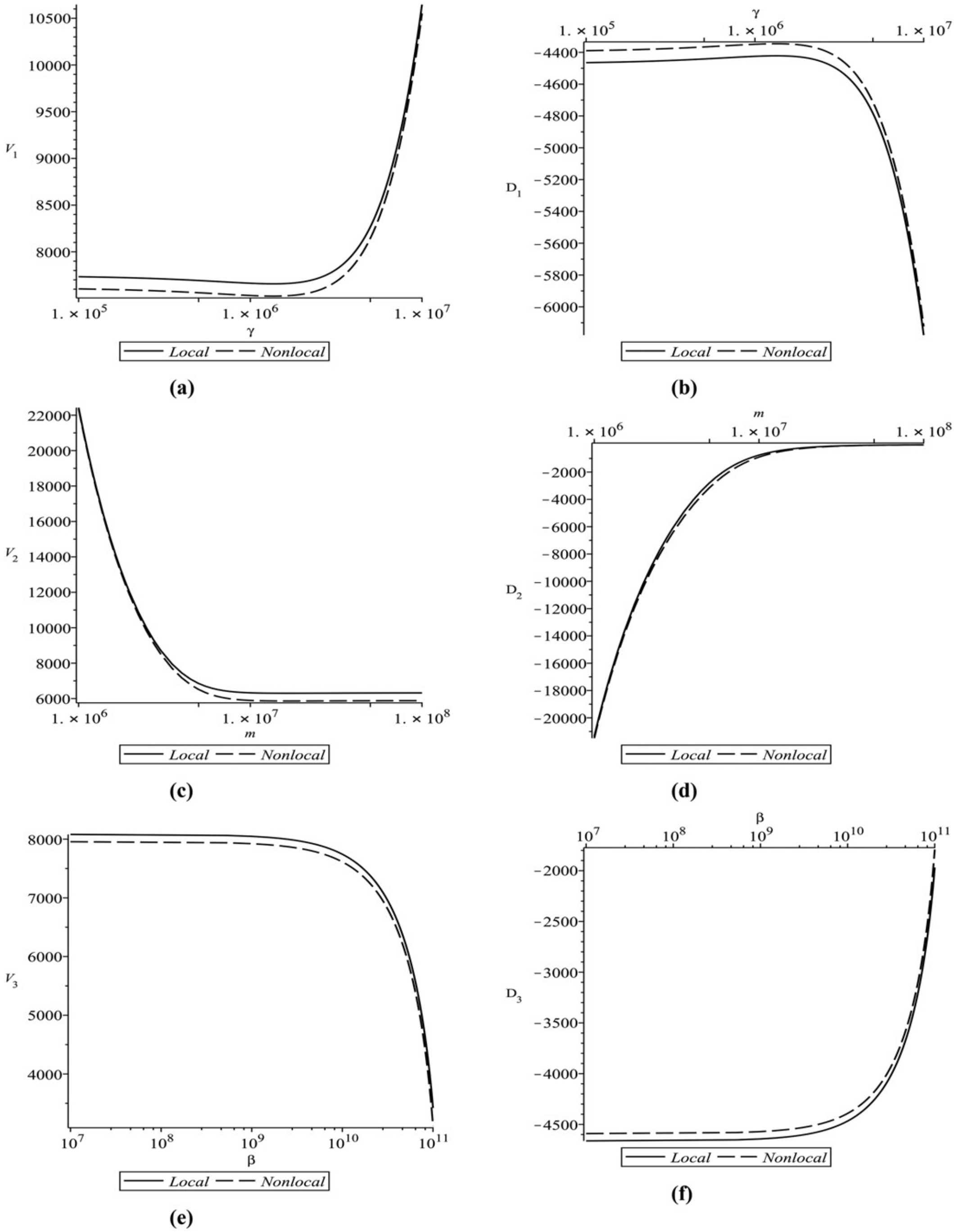

Figure 4. Effects of various coupling parameters (β, γ, m) on wave speeds and corresponding damping coefficients.

- The effect of *thermovoid coupling* parameter m can be viewed through Figure 4c, d for wave speed V_2 and corresponding damping coefficient D_2 for local and nonlocal medium. It is observed that larger magnitudes of m diminish the magnitudes in both V_2 and D_2.
- The influence of *elastovoid coupling* parameter β can be noticed from Figure 4e, f for wave speed V_3 and corresponding damping coefficient D_3 in local as well as nonlocal

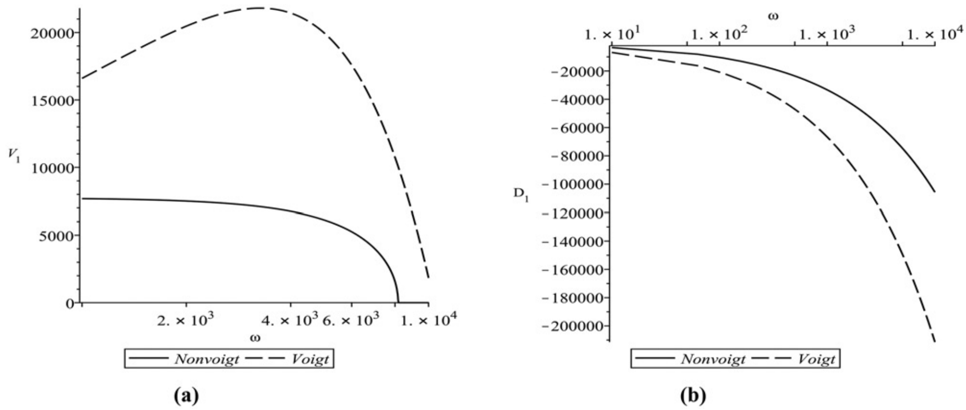

Figure 5. Comparison of wave speed V_1 and the corresponding damping coefficient D_1 with respect to ω for Voigt ($\tau = 0$) and non-Voigt ($\tau \neq 0$) material.

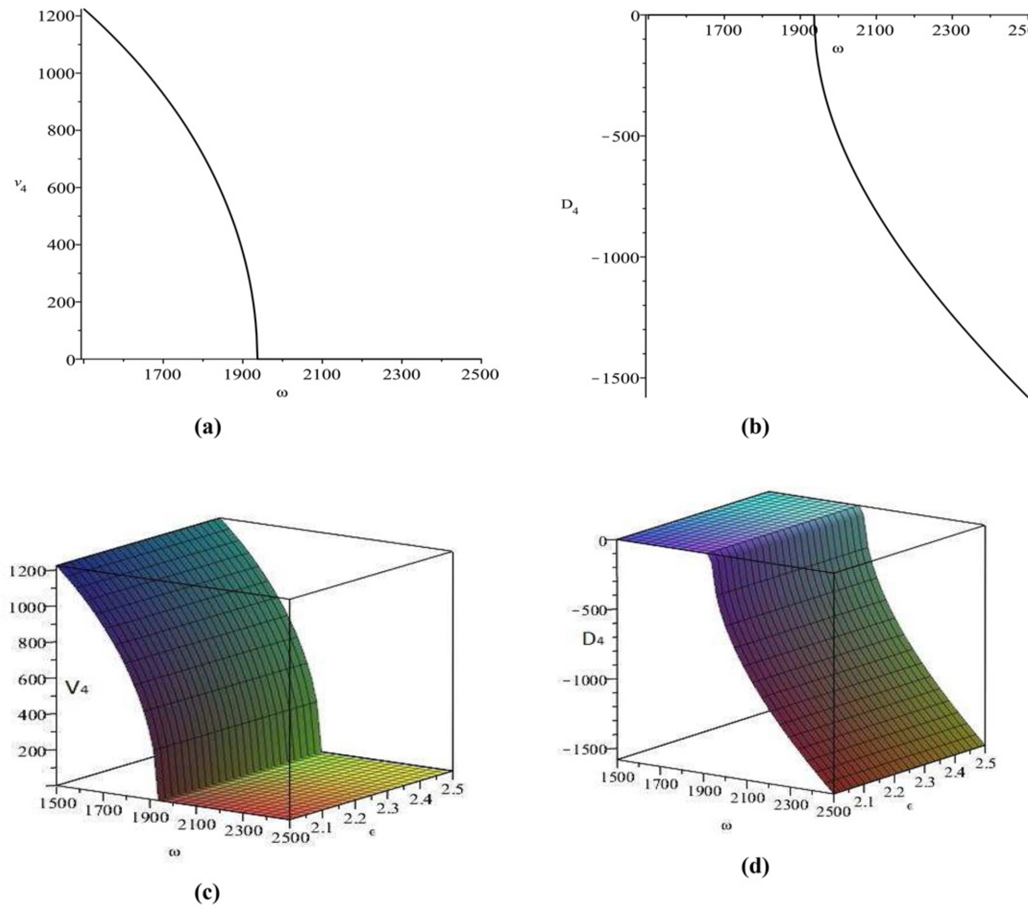

Figure 6. Variations of the wave speed V_4 and the corresponding damping coefficient D_4 against ω and ϵ.

medium. It is depicted that larger magnitude of β reduces the magnitudes in both V_3 and D_3.

Figure 5a, b is drawn to analyze the effect of Voigt parameter τ on the wave speed and corresponding damping coefficient and we note the following facts:

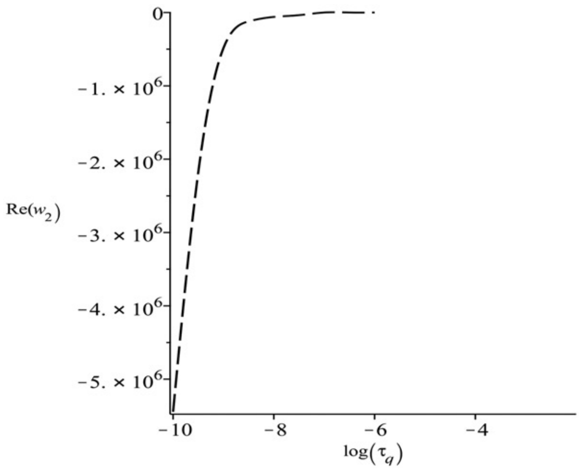

Figure 7. Variation of $\Re(w_2)$ of Eq. (30) with respect to τ_q (logarithmic scale).

- The wave speed in Voigt material is larger when compared to that in case of non-Voigt material (see Figure 5a).
- In Figure 5b, we observed more damping in time in case of Voigt material when compared to that in case of non-Voigt material.

Figure 6a, b displays the wave speed and corresponding damping coefficient of transverse wave and we make the following remarks.

- From this figure, we note that the transverse wave is dispersive and undamped in time in the range: $0 \leq \omega < 1936.49$, beyond which the transverse wave is not a propagating wave. This is the verification of a result pointed out theoretically in the text with the numerically obtained result. It can also be verified that $\omega \equiv \omega_c = 1936.49$ is correct which is clearly reflected through Figure 6a, b.
- The cutoff frequency ωc varies inversely with ϵ as ct is constant for a particular material. To exhibit this phenomena, we present two more graphs through Figure 6c, d. Figure 6b, d reflects the fact that the damping in time of the amplitude of transverse wave decays exponentially, a result pointed out theoretically in the text.

At last, in order to make a comparison graphically with a result presented by Chirita, Ciarletta, and Tibullo [23], Figure 7 has been plotted to show the behavior of the real part of the root w_2 of Eq. (30) versus τ_q (logarithmic scale) as in [23]. We conclude from this figure that our numerically computed results agree with the existing literature by Chirita, Ciarletta, and Tibullo [23] up to a satisfactory level. The pattern of the curve $\Re(w_2)$ is qualitatively same with the curve obtained in Figure 1 by Chirita, Ciarletta, and Tibullo [23] with small deviation in magnitudes, as we have used different values of the material parameters from those considered in [23].

Conclusions

The propagation of time harmonic plane waves in an infinite nonlocal dual-phase-lag thermoelastic medium with voids has been explored. The elastic nonlocality in the thermal field has been accounted through dual-phase-lag heat conduction law (5). From this study, we can infer the following important facts:

1. Four different types of time harmonic plane waves propagating with distinct wave velocities travel in the medium considered consisting of three sets of coupled dilatational waves and an independent transverse wave.
2. The sets of coupled longitudinal waves are found to be dispersive in nature and damping in time. The transverse wave is dispersive and undamped up to the cutoff frequency ω_c, beyond which the wave becomes undamped standing wave.
3. All the existing waves are found to be influenced by the nonlocality of the medium. The speed of the transverse wave is reduced due to the presence of nonlocality in the medium.
4. The waves propagating with speeds V_i $(i = 1, 2, 3)$ are influenced by the void parameters as well as thermal parameters, whereas the wave propagating with velocity V_4 is independent of the these parameters.
5. Wave speeds are always remaining larger in case of local medium as compared to the nonlocal medium.

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